Math 371 **Name:** ______________________

Spring 2019

Practice Midterm 2

4/3/2019

Time Limit: 80 Minutes ID ______________________

"My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this"

Signature ______________________

This exam contains 10 pages (including this cover page) and 9 questions.
Total of points is 108.

- Check your exam to make sure all 10 pages are present.
- You may use writing implements and a single handwritten sheet of 8.5"x11" paper.
- NO CALCULATORS.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Good luck!

Grade Table (for teacher use only)

Question	Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	12	
8	12	
9	12	
Total:	108	

1. (12 points) State the definition of an ideal of a ring. Find all the ideals in $\mathbb{Z}/6\mathbb{Z}$.

2. (12 points) Find the units in $\mathbb{Z}/9\mathbb{Z}$.

3. (12 points) Is $(i + 4)$ a maximal ideal in $\mathbb{Z}[i]$? Why?

4. (12 points) What are the maximal ideals of $\mathbb{C}[x, y]/(xy, (x-2)(y-1))$?

5. (12 points) Find the kernel of the homomorphism $\mathbb{C}[x, y] \to \mathbb{C}[t]$ determined by $x \mapsto t^2 + t, y \mapsto t - 1$.

6. (12 points) Give an example of irreducible polynomial $f(x)$ of degree 2 in $\mathbb{F}_3[x]$. Use $f(x)$ to construct an example of a field consisting of 9 elements.

7. (12 points) State the definition of prime element in an integral domain R. Find all the prime elements in $\mathbb{C}[t]$

8. (12 points) Prove that $\mathbb{Z}[i]/(3)$ is a field.

9. (12 points) Let $f = x^3 + x^2 + x + 1$ and let α denote the residue of x in the ring $R = \mathbb{Z}[x]/(f)$. Express $(\alpha^4 + \alpha)(\alpha + 1)$ in terms of the basis $(1, \alpha, \alpha^2)$ of R.